

Quantum computing based on space states without charge transfer

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An implementation of a quantum computer based on space states in double quantum dots is discussed. There is no charge transfer in qubits during calculation, therefore, uncontrollable entanglement between them due to long-range Coulomb interaction is suppressed. Other plausible sources of decoherence caused by interaction with phonons and gates could be substantially suppressed in the structure too. We also demonstrate how all necessary quantum logic operations, initialization, writing, and read-out could be carried out in the computer.

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I. INTRODUCTION

Almost all recent experimental realizations of quantum computation accomplished on several solid state qubits and even theoretical proposals of new devices were based on spin states. Much cited papers of Kane [1] and DiVincenzo et al. [2, 3, 4] just concerned nucleus or electron spin encoding in solid state implementation of a quantum computer (QC). The attractiveness of spin states in solid state QC was mainly caused by quite long decoherence time: hours for a nucleus spin and milliseconds for electron spin seem attainable [1, 4].

Nevertheless, space or charge states for quantum encoding do not seem less prospective although two main disadvantages were commonly mentioned [3]. Firstly, much less decoherence time was expected for charge states. Secondly, charge transfer results in uncontrollable interaction between even distant qubits due to long-range Coulomb forces. However, owing to the pioneer papers [5, 6, 7] now it is clear that for fairly small energy gap between different qubit states decoherence caused by phonons might be very weak. This has opened trends in quantum computers based on double quantum dots. Recently they were intensively studied in experiments [8, 9, 10]. The similar structures based on double donor atoms in silicon were proposed in Ref. [11].

Here we withdraw the second shortcoming of a space state based QC discussing a construction [12, 13], where no charge transfer occurs during computation. Besides, the computation may be much faster than that in a spin based QC. Moreover, the read-out of charge states is looking easier and faster than that of spin states.

II. QUBIT AND ITS OPERATION

A qubit is implemented in two double quantum dots (DQD) (Fig. 1). A DQD consists of a pair of quantum

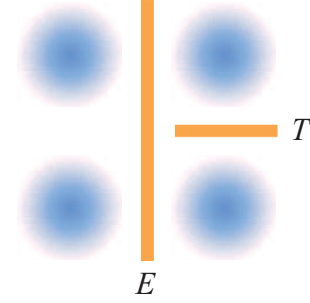


FIG. 1: Qubit structure.

dots with a single electron. The electrode operates on the strength of exchange interaction between electrons in DQDs. The electrode varies tunneling coupling between quantum dots constituting a DQD.

We designate two lowest states of an electron in a DQD as $|+\rangle$ and $|-\rangle$. The state

$$|+\rangle = \frac{1}{\sqrt{2}}(\psi(r-r_1) + \psi(r-r_2)) \quad (1)$$

is symmetric and the state

$$|-\rangle = \frac{1}{\sqrt{2}}(\psi(r-r_1) - \psi(r-r_2)) \quad (2)$$

is anti-symmetric (Fig. 2). Here r_1 and r_2 are coordinates of the center of the first and second quantum dots, $\psi(r)$ is an electron wave function in a dot, it decays as $e^{-r/a}$ outside the dot, the magnitude a of depends on a potential barrier height which may be controlled by a voltage on gate T .

Two basic computational states of the qubit composed of two DQDs are

$$|0\rangle = |+-\rangle, \quad |1\rangle = |-+\rangle. \quad (3)$$

In these states an electron in the first DQD is in symmetric state and another electron in the second DQD is in anti-symmetric state, and vice versa. As an exchange

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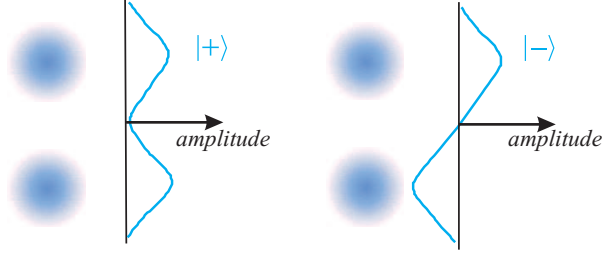


FIG. 2: Wave functions of an electron in a DQD (two lowest states).

interaction is to be involved later, one should take into account the Fermi-Dirac statistics of electrons. When two electrons are in the triplet spin state (a total spin $S = 1$), i.e., have a symmetric spin configuration, their space wave function is anti-symmetric with respect to permutation. Hereafter we assume an overall spin polarization of electrons in the system and, therefore, instead of states (3) one should use the following states:

$$|0\rangle = \frac{1}{\sqrt{2}}(|+1-2\rangle - |+2-1\rangle), \quad (4)$$

$$|1\rangle = \frac{1}{\sqrt{2}}(|-1+2\rangle - |-2+1\rangle), \quad (5)$$

where the indices 1 and 2 correspond to the first and second electron, respectively. Any superpositional state of a qubit

$$\frac{a|0\rangle + b|1\rangle}{\sqrt{a^2 + b^2}} \quad (6)$$

is created with the help of gate electrodes. Here a and b are arbitrary complex numbers. Thus qubit states belong to the Hilbert sub-space produced by orthogonal states $|+-\rangle$ and $| - + \rangle$.

Worth noting no charge transfer is required to produce any of the states (6). It can be easily confirmed that for any numbers a and b the probability to find an electron in any quantum dot is always equal to 0.5. This is just a reason why a qubit based on a pair of DQDs is required instead of a qubit based on a single DQD.

Hereafter, the unitary transformations of a qubit are described by the Hamiltonian in matrix presentation in basis of states (4) and (5)

$$\mathcal{H} = A \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + P \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (7)$$

where the factors A and P depend upon a voltage applied to gates E and T , respectively. The associated unitary time evolution is determined by the operator

$$\hat{U}(t) = \hat{T} \exp \left[-\frac{i}{\hbar} \int_0^t \mathcal{H}(\tau) d\tau \right], \quad (8)$$

where \hat{T} is a time ordering operator.

The amplitude shift can be realized by applying voltage to electrode E which operates upon the strength of exchange interaction between electrons located in surrounding DQDs (Fig. 1). The applied positive voltage diminishes the potential barrier height and augments the wave function overlap. It leads to enhancement of exchange interaction. In particular, an amplitude flip resulting in a transition of a state $|0\rangle$ into a state $|1\rangle$ can be performed by means of appropriate pulse amplitude and duration. An exchange interaction is based on Coulomb interaction between electrons

$$\hat{U}_C(r_1, r_2) = \frac{e^2}{\kappa|r_1 - r_2|}, \quad (9)$$

where r_1 and r_2 are respectively the coordinates of first and second electron, κ is a permittivity of an environment. The direct calculation of matrix elements of operator \hat{U}_C for transitions from basic qubit states $|+-\rangle$ and $| - + \rangle$ to complimentary states $|++\rangle$ or $|--\rangle$ gives zero in contrast with transitions between basic states. Obviously, beforehand the states $|++\rangle$ and $|--\rangle$ should be also presented in the form like (4)-(5). Moreover, the transitions to these states are prohibited by symmetry conservation. Indeed, $|++\rangle$ and $|--\rangle$ states have additional symmetry with respect to inversion over E -electrode axis. The Coulomb operator \hat{U}_C can not break this symmetry.

A phase shift of a qubit is operated by the gate electrode T acting on a tunneling coupling between the associated quantum dots. When a positive potential is applied tunneling between constituent quantum dots is reinforced and the energy difference $\Delta\varepsilon$ between $|+\rangle$ and $|-\rangle$ states of this DQD becomes greater. It results in steadily rising phase difference between $|+\rangle$ and $|-\rangle$ states. In particular, for a proper impulse duration and amplitude the phase difference achieves π , i.e., a phase flip occurs. It should be outlined that no tunneling can happen during a phase shift operation as it needs a definite bias between quantum dots (see for the section "Read-out").

III. TWO-QUBIT OPERATIONS

The electrodes of E -type placed between adjacent qubits make possible to perform two-qubit operations. The most simple for realization is a SWAP operation, that is, an exchange of states between neighbor qubits. This operation is fulfilled merely by sequential application of voltage pulses to electrodes E resulting in exchange of states between surrounding DQDs. Thus, the SWAP operation between neighbor qubits can be easily represented by the formula $SWAP = \hat{E} \cdot NOT_1 \cdot NOT_2 \cdot \hat{E}$, where \hat{E} is exchange operation with the help of E -electrode located between the first and the second qubits, NOT_1 and NOT_2 are NOT operations in the first and in

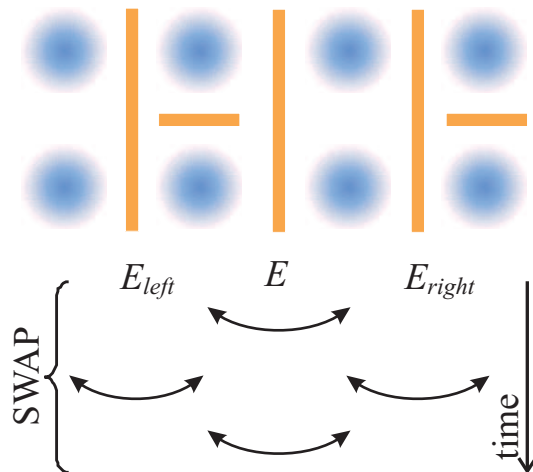


FIG. 3: SWAP realization.

the second qubits respectively (Fig. 3). The SWAP operation allows to move any qubit along a chain and thus put in contact any pair of qubits.

Realization of CNOT is based on the square root of the SWAP operator, which is given by the expression $SWAP = \hat{E} \cdot \sqrt{NOT_1} \cdot \sqrt{NOT_2} \cdot \hat{E}$. The definite performance of CNOT operation is presented in Appendix. Other 2-qubit logic operations could be also composed in a similar way. The rate of 1-qubit and 2-qubit operations depend on the strength of exchange interaction and tunneling coupling augmented by voltage applied to electrodes. The offered construction permits, in principle, a clock speed up to 1THz compared to 1 GHz for electron spin based QC [4] or 75 kHz for nucleus spin based QC [1].

IV. INITIALIZATION OF A COMPUTER

One way of initial state creation in the QC is to input electrons in a necessary state through the end of preliminary empty qubit chain. An electron can be supplied to the extreme DQD by some single-electron device, for instance, a turnstile. A symmetric state $|+\rangle$ can be prepared by applying a fairly high voltage to an electrode T to make a sufficient energy difference between $|+\rangle$ and $|-\rangle$ states. The first one is a ground state of an electron in a DQD. Thus for fairly large energy difference between ground $|+\rangle$ and excited $|-\rangle$ states with respect to a thermal energy kT the occupancy of the upper state can be done negligible. A state $|+\rangle$ can be easily inverted into a state $|-\rangle$ (if required) when a DQD is biased by means of some additional electrode placed near one quantum dot. Afterwards, an electron with a formed state can be pushed from the starting DQD to any DQD in a chain by voltage pulses successively applied to E -electrodes along the path. In this way one can pump electrons one by one along a qubit chain and fill all DQDs.

Another possibility of initialization implies the forma-

tion of necessary state immediately in a particular qubit by means of foregoing procedure.

All electrons in the proposed QC should be spin-polarized as an exchange interaction depends on a spin configuration. Two opportunities seem like plausible. Obviously, it could be done by external magnetic field and cooling. Another way is to supply electrons from a ferromagnetic contact.

There is a possibility to speed up $|+\rangle$ or $|-\rangle$ states creation even under condition $\Delta\varepsilon < kT$ when a thermal relaxation is evidently ineffective. The procedure merely looks like the reversed read-out considered in the next section.

V. READ-OUT

To read out the information accumulated in the resultant register of a QC one should measure the state of DQDs. The following procedure is proposed. The voltage applied to the gate T makes an essential energy gap $\Delta\varepsilon$ between $|+\rangle$ and $|-\rangle$ states. The mean energy of a system when electron occupies one quantum dot is situated just in the middle of the gap. When this DQD is biased by a voltage $\frac{\Delta\varepsilon}{2e}$ by means of outer electrode placed near one quantum dot resonant tunneling of an electron to a quantum dot occurs. To what dot of two depends on initial state. Surely, the state when an electron is located in one quantum dot is not an eigen state of a system. Actually on applying a bias the electron begins to oscillate between quantum dots. But what dot it visits first depends on the initial state of a DQD, whether it is $|+\rangle$ or $|-\rangle$. One could switch off T -voltage, i.e., tunnelling, right at the moment when electron is located in one quantum dot. Measuring a conductance of a quantum wire placed nearby one can easily distinguish what quantum dot contains the electron. Moreover, a current-voltage curve of a single wire placed along a resultant qubit chain could, in principle, provide the whole information about its charge state [14].

VI. DECOHERENCE: INTERACTION WITH GATES AND PHONONS

The weaker is decoherence the longer is a fault tolerant time of a QC. The decoherence in our QC circuit arises when a state of some DQD is altered from $|+\rangle$ to $|-\rangle$, or vice versa. Two comprehensible sources of decoherence in the system are voltage fluctuations and phonons. One more source mentioned recently is the Joule loss in a nearby metallic electrode caused by displacement of charge inside a qubit [15, 16]. This kind of decoherence is absent in the QC under consideration because no charge transfer occurs.

Voltage fluctuations were discussed in [1]. There resonant transitions between states were induced by associated spectrum component of a voltage noise. However, a

more dangerous mechanism is a phase diffusion of a qubit. It takes place when voltage fluctuations shift phase of a qubit slightly in random way. In some time that leads to a definite phase shift. In the present construction voltage fluctuations on electrodes T and E can not affect transitions between $|+\rangle$ and $|-\rangle$ states owing to symmetry of the structure. One more advantage relevant to offered implementation is that manipulation with some qubit does not perturbs the state of neighbor ones.

However, optical gates to control a potential barrier transparency [17, 18, 19] could be used instead of voltage gates. Extremely strong dependence of a photo-stimulated tunneling on a beam polarization permits a precise addressing to a definite qubit [19].

As for the decoherence induced by phonons, the cheerful results were obtained in [5, 6, 7]. They much contradict with intuitive expectations. Here we restrict the discussion by rather qualitative consideration. The states $|+\rangle$ and $|-\rangle$ have very small energy difference $\Delta\varepsilon$ in an idle qubit. The same is the energy of an acoustic phonon ε required to enforce a transition between these states. The lower is the energy the smaller is the matrix element of these transitions. One reason is a long wave length of a relevant phonon. In [6] a probability τ^{-1} of spontaneous emission of acoustic phonons in DQD was calculated. There were obtained dependencies $\tau \sim \Delta\varepsilon^{-5}$ for deformation acoustic phonons and $\tau \sim \Delta\varepsilon^{-3}$ for piezoelectric acoustic phonons. The latter do not exist in Si

but dominate in compounds. When two $GaAs/AlGaAs$ quantum dots constituting a DQD are separated by a distance $r = 22 \text{ nm}$ the values $\tau \sim 10^{-2} \text{ s}$ for piezoelectric acoustic phonons and $\tau \sim 10^{-6} \text{ s}$ for deformation acoustic phonons are attainable. The stimulated emission and absorption, and multi-phonon processes were beyond the frame of [5, 6, 7]. However, they may be significant when $\Delta\varepsilon < kT$. To roughly evaluate the stimulated emission one should multiply the above magnitude of τ by a small factor $\frac{\Delta\varepsilon}{kT}$ originating from the Bose-Einstein statistics for $\Delta\varepsilon \ll kT$, $n(\Delta\varepsilon) = [e^{\Delta\varepsilon/kT} - 1]^{-1} \approx \frac{kT}{\Delta\varepsilon}$. Nevertheless, the probability to emit or to absorb a phonon still drops with energy difference $\Delta\varepsilon$. It looks amazing as if one could sustain the coherence between states separated by energy less than kT for an arbitrary long time.

In our opinion, in reality two-phonon processes set a limit for decoherence time. During transition from a state $|+\rangle$ to a state $|-\rangle$ one phonon is emitted and another absorbed. The energy of phonons involved is about kT , that is, independent on energy split $\Delta\varepsilon$ between states $|+\rangle$ and $|-\rangle$.

Here we calculate the probability of transitions from state $|+-\rangle$ to states $|--\rangle$, $|++\rangle$, or $|+-\rangle$ due to absorption of a phonon with a wave vector \mathbf{q} and emission of a phonon \mathbf{q}' in the second order of the perturbation theory. The probability of the transition from the state $|+-\rangle$ to an arbitrary final state $|f\rangle$ is

$$W(|+-\rangle \rightarrow |f\rangle) = \frac{2\pi}{\hbar} \sum_{\mathbf{q}} \sum_{\mathbf{q}'} |F_{\mathbf{q}}|^2 |F_{\mathbf{q}'}|^2 (n(\hbar\omega_{\mathbf{q}}) + 1) n(\hbar\omega_{\mathbf{q}'}) \times \frac{\left| \sum_z \langle f | e^{i\mathbf{q}'\mathbf{r}} | z \rangle \langle z | e^{-i\mathbf{q}\mathbf{r}} | +-\rangle \right|^2}{(\varepsilon_z - \varepsilon_{+-} - \hbar\omega_{\mathbf{q}})^2} \delta(\varepsilon_f + \hbar\omega_{\mathbf{q}'} - \varepsilon_{+-} - \hbar\omega_{\mathbf{q}}). \quad (10)$$

Here $F_{\mathbf{q}}$ is electron-phonon coupling, $\omega_{\mathbf{q}}$, $\omega_{\mathbf{q}'}$ are phonons frequencies, ε_{+-} and ε_f are energies of initial and final states, the summation over intermediate states $|z\rangle$ is made. Because of a high energy of upper states only the states $|+-\rangle$, $|--\rangle$, $|++\rangle$, and $|+-\rangle$ can be put into account as intermediate ones. As far as phonons with energy $\hbar\omega_{\mathbf{q}} \approx kT \gg \Delta\varepsilon$ mostly contribute to the probability the denominator $(\varepsilon_z - \varepsilon_{+-} - \hbar\omega_{\mathbf{q}})^2$ can be reduced to $(kT)^2$. Then the evaluation of probability gives the dependence $W \sim T^6$ for deformation acoustic phonons and the dependence $W \sim T^2$ for piezoelectric acoustic phonons. Hence, just two-phonon processes set a limit of decoherence rate when the energy separation is decreasing to zero.

VII. CONCLUSIONS

In conclusion, an implementation of a space state based quantum computer without charge transfer is discussed. A qubit consists of four quantum dots with two electrons. Evolution of the system is controlled with gate voltages operating on tunneling coupling and strength of exchange interaction. As there is no charge transfer during calculation, therefore, uncontrollable entanglement between qubits due to long-range Coulomb forces is suppressed. The decoherence caused by metallic electrodes is much diminished in the structure. Phonon-induced decoherence is limited by two-phonon processes. High-speed computation and long fault tolerant time look as feasible.

Acknowledgments

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APPENDIX: TWO-QUBIT OPERATIONS

Here we consider 2-qubit operations in detail. The initial state of pair of qubits is a linear combination of the following basic states:

1 st qubit	2 nd qubit	basis state	(A.1)
+-	+-	(1000) ^T	
+-	-+	(0100) ^T	
-+	+-	(0010) ^T	
-+	-+	(0001) ^T	

Unfortunately, this basis does not cover all states originating during SWAP operation. Indeed, if we apply voltage to electrode placed between qubits in question, then state will become which does not belong to the original sub-space. Thus, the sub-space should be extended as follows:

1 st qubit	2 nd qubit	basis state	(A.2)
+-	+-	(100000) ^T	
++	--	(010000) ^T	
+-	-+	(001000) ^T	
-+	+-	(000100) ^T	
--	++	(000010) ^T	
-+	-+	(000001) ^T	

Just this requirement makes the realization of CNOT in the structure under consideration non-trivial. Hereafter, we represent all operations by matrices. The NOT operations in the first and in the second qubit are

$$NOT_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}, \quad (A.3)$$

$$NOT_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}. \quad (A.4)$$

The SWAP operation between adjacent DQDs in neighbor qubits is

$$\hat{E} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \quad (A.5)$$

The matrix of SWAP operation between qubits reads

$$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (A.6)$$

Fig. 3 illustrates the plausible way to construct SWAP. Indeed,

$$\hat{E} \cdot NOT_1 \cdot NOT_2 \cdot \hat{E} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} = SWAP. \quad (A.7)$$

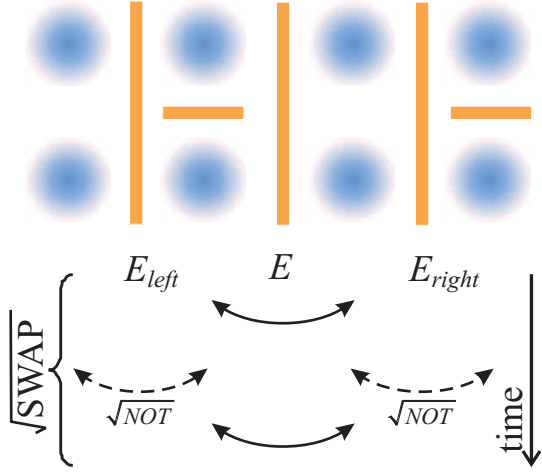
We should notice that matrices NOT_1 and NOT_2 commute, i.e., $NOT_1 \cdot NOT_2 = NOT_2 \cdot NOT_1$. Moreover, $\hat{E} \cdot \hat{E} = \hat{E}^2 = \hat{1}$.

Square root of NOT operation can be performed as NOT operation with a half pulse duration. The matrices of such operations are

$$\sqrt{NOT_1} = \frac{1}{\sqrt{2i}} \begin{pmatrix} 1 & 0 & 0 & i & 0 & 0 \\ 0 & \sqrt{2i} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & i \\ i & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2i} & 0 \\ 0 & 0 & i & 0 & 0 & 1 \end{pmatrix}, \quad (A.8)$$

$$\sqrt{NOT_2} = \frac{1}{\sqrt{2i}} \begin{pmatrix} 1 & 0 & i & 0 & 0 & 0 \\ 0 & \sqrt{2i} & 0 & 0 & 0 & 0 \\ i & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & i \\ 0 & 0 & 0 & 0 & \sqrt{2i} & 0 \\ 0 & 0 & 0 & i & 0 & 1 \end{pmatrix}. \quad (A.9)$$

The commutativity of matrices $\sqrt{NOT_1}$ and $\sqrt{NOT_2}$ could be easily checked out. Then

FIG. 4: \sqrt{SWAP} realization.

$$\begin{aligned}
SWAP &= \hat{E} \cdot NOT_1 \cdot NOT_2 \cdot \hat{E} \\
&= \hat{E} \cdot \sqrt{NOT_1} \sqrt{NOT_1} \cdot \sqrt{NOT_2} \sqrt{NOT_2} \cdot \hat{E} \\
&= \hat{E} \cdot \sqrt{NOT_1} \sqrt{NOT_2} \cdot \sqrt{NOT_1} \sqrt{NOT_2} \cdot \hat{E} \\
&= \hat{E} \cdot \sqrt{NOT_1} \sqrt{NOT_2} \cdot \hat{E} \hat{E} \cdot \sqrt{NOT_1} \sqrt{NOT_2} \cdot \hat{E} \\
&= \left(\hat{E} \cdot \sqrt{NOT_1} \sqrt{NOT_2} \cdot \hat{E} \right)^2. \tag{A.10}
\end{aligned}$$

Consequently,

$$\begin{aligned}
\sqrt{SWAP} &= \hat{E} \cdot \sqrt{NOT_1} \sqrt{NOT_2} \cdot \hat{E} \\
&= \frac{1}{2i} \begin{pmatrix} 2i & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & i & i & -1 & 0 \\ 0 & i & 1 & -1 & i & 0 \\ 0 & i & -1 & 1 & i & 0 \\ 0 & -1 & i & i & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2i \end{pmatrix} \tag{A.11}
\end{aligned}$$

The scheme of square root of SWAP is shown in Fig. 4. The dimensionality of all matrices is 6×6 . This property leads to the particular form of the operator of controlled phase shift

$$\hat{\Pi} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}. \tag{A.12}$$

As far as matrices 4×4 are concerned, XOR operation is

$$\begin{aligned}
&\left(\hat{Z}_1(\pi/2) \otimes \hat{Z}_2(-\pi/2) \right) \cdot \sqrt{SWAP} \\
&\times \left(\hat{Z}_1(\pi) \otimes \hat{1}_2 \right) \cdot \sqrt{SWAP}, \tag{A.13}
\end{aligned}$$

where \hat{Z} is the phase shift gate [2]. In a similar way, a direct calculation shows that in our case

$$\begin{aligned}
\hat{\Pi} &= \left[\left(\hat{Z}_1(\pi/2) \otimes \hat{Z}_2(-\pi/2) \right) \cdot \sqrt{SWAP} \right]^2 \\
&\times \left(\hat{Z}_1(\pi) \otimes \hat{1}_2 \right) \cdot \sqrt{SWAP}. \tag{A.14}
\end{aligned}$$

Eventually, CNOT operation looks like

$$CNOT = \left(\hat{1}_1 \otimes \hat{H}_2 \right) \cdot \hat{\Pi} \cdot \left(\hat{1}_1 \otimes \hat{H}_2 \right), \tag{A.15}$$

where \hat{H} is Hadamard's transformation:

$$\hat{1}_1 \otimes \hat{H}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix}. \tag{A.16}$$

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